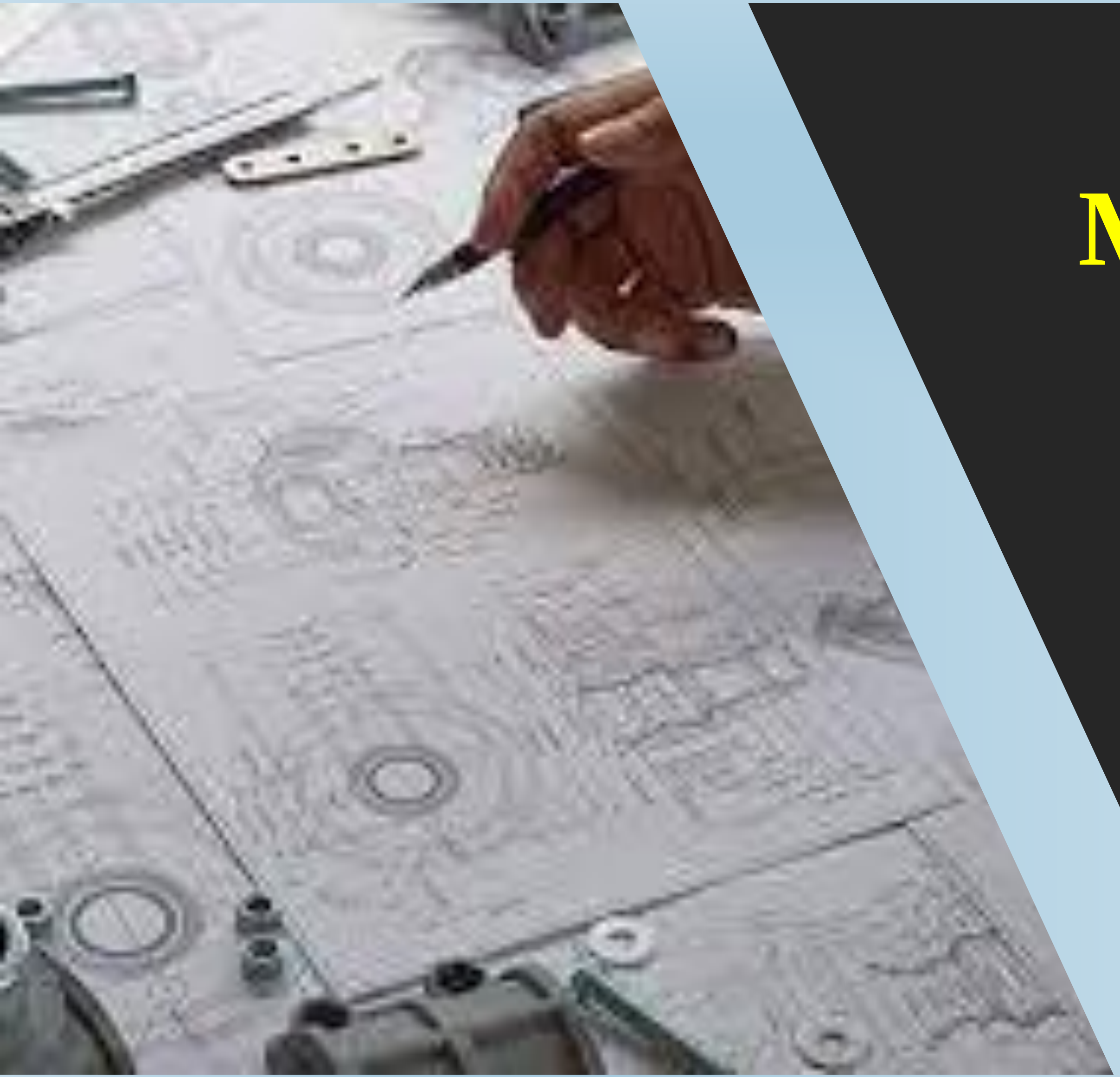
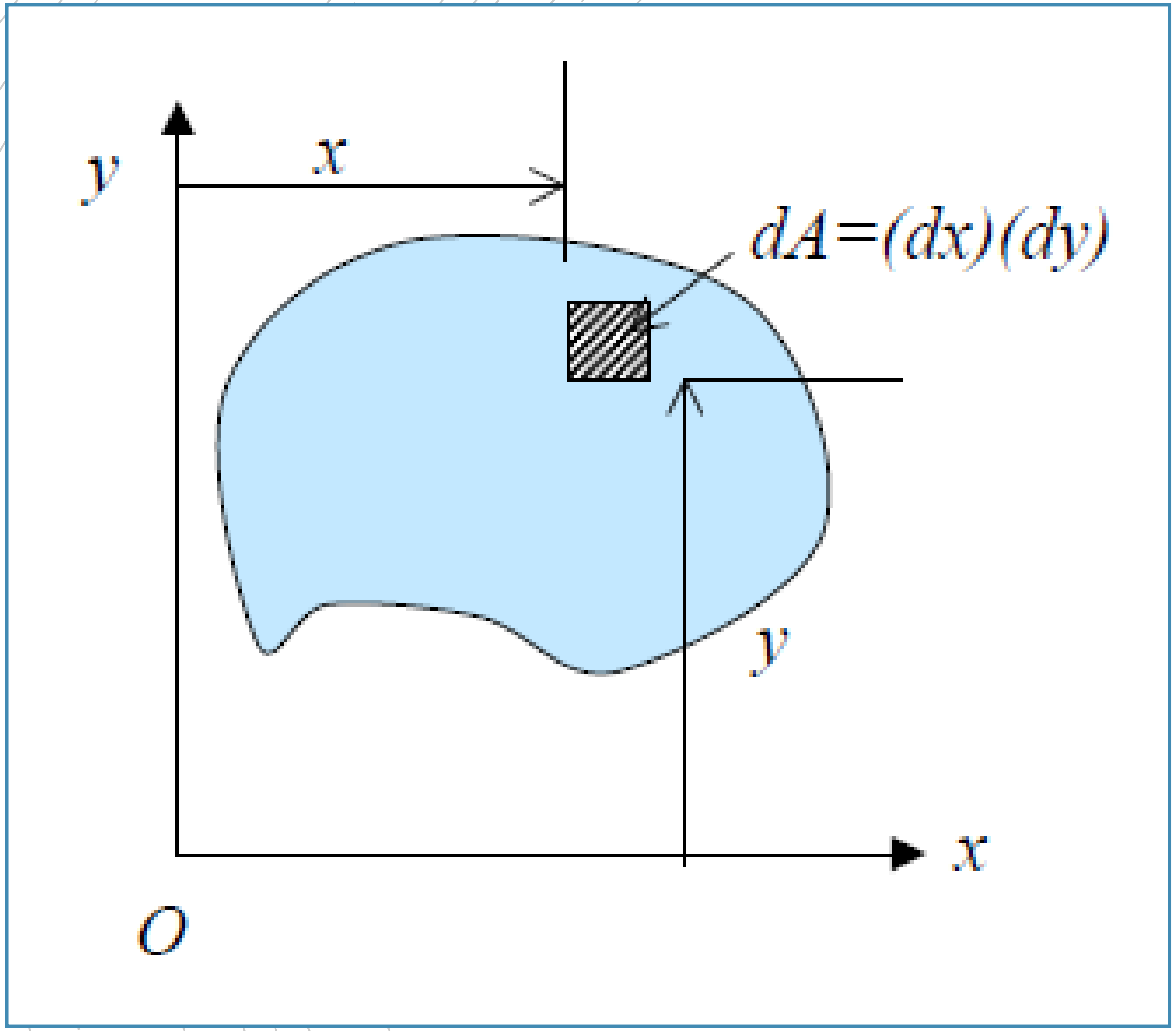


MECHANICS

Lecture No.2

Moment of Inertia

د. محمد سعد



Moment of Inertia

- The moment of inertia can be considered as a shape factor which indicated how the material is distributed about the center of gravity of the cross-section. **It's also defined as the capacity of a cross-section to resist bending**. It is usually quantified in m4 or kg.m2. So, the moment of inertia has a significant effect on the structural behavior of construction elements. The formulas used for determining the moment of inertia are:

$$I_x = \sum y^2 \, . \, dA$$

$$I_Y = \sum X^2 \, . \, dA$$

I_x: Moment of inertia about x axis.

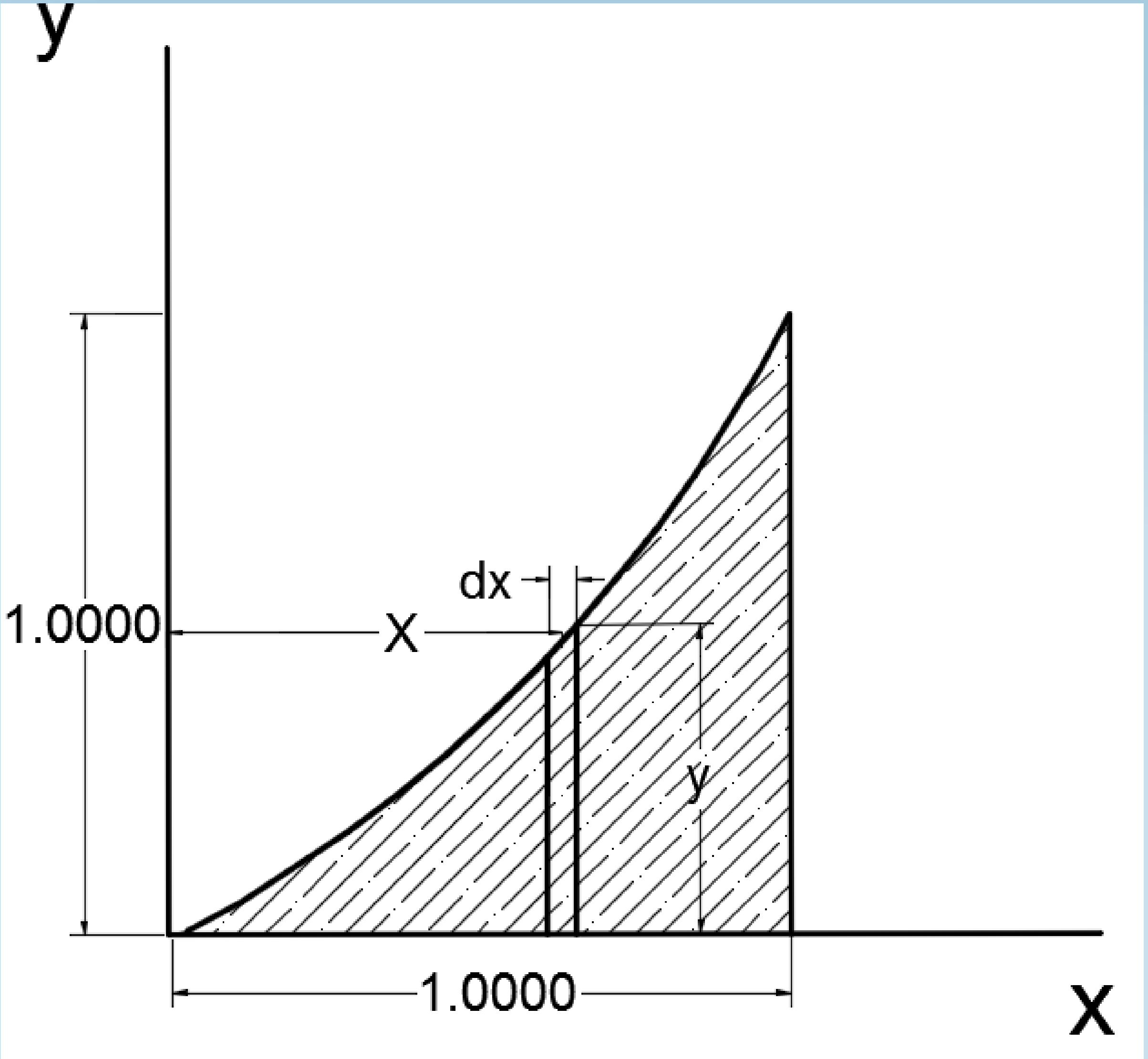
I_Y: Moment of inertia about y axis.

Note:

The formula above is used to calculate the moment of inertia directly when taking a parallel slice of the axis which required determine moment of inertia about it.

Example (1): Determine the moment of inertia of the shaded area in the fig. (1) With y-axis the equation of the curve is $x^3 = y^2$

Fig. (1)



$$\int dA = \int_0^1 y.d_x$$

$$\int dI_y = \int_0^1 x^2.dA$$

$$y = x^{\frac{3}{2}}$$

$$\int dI_y = \int_0^1 x^2.y.d_x$$

$$\int dI_y = \int_0^1 x^2.x^{\frac{3}{2}}.d_x = \int_0^1 .x^{\frac{7}{2}}.d_x$$

$$I_y = \left[\frac{x^{\frac{9}{2}}}{\frac{9}{2}}\right] = 0.222\, m^4$$

Example (2): Determine the moment of inertia of the shaded area in the fig. (2) With y-axis the equation of the curve is $y = 2 - 2x^3$

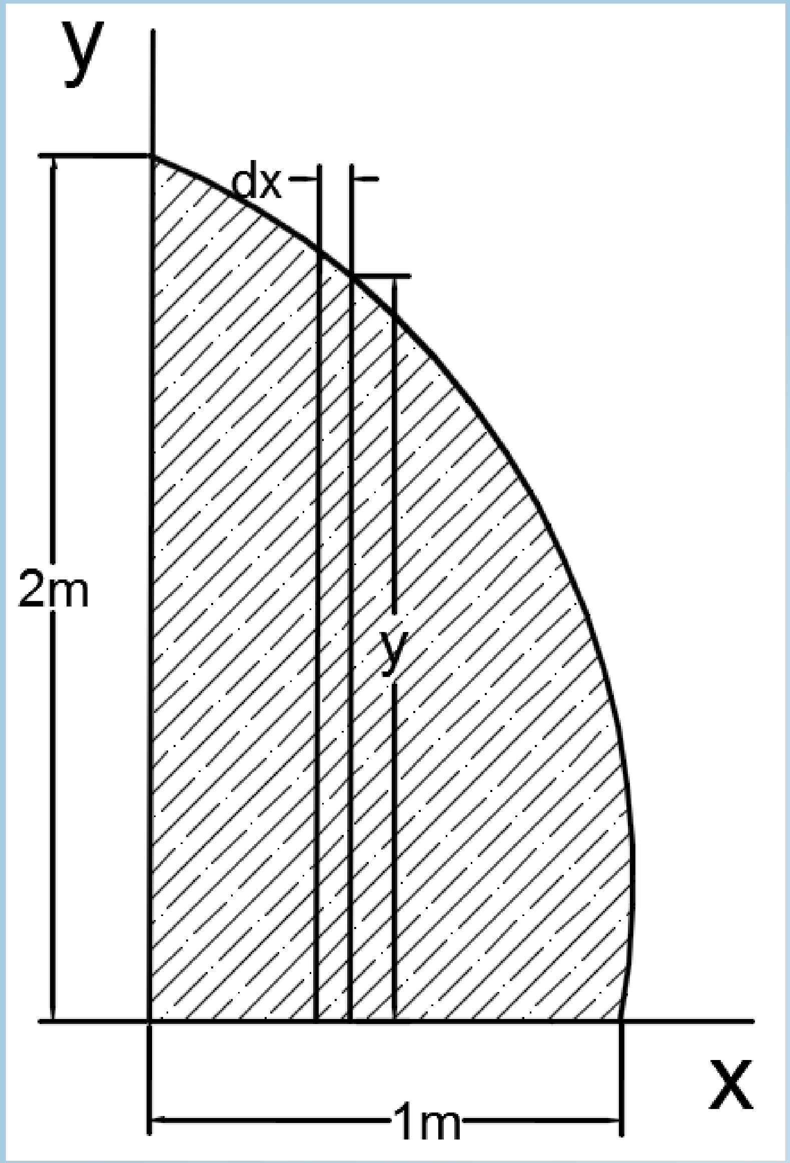


Fig. (2)

$$dI_y = \int_0^1 x^2 . dA$$

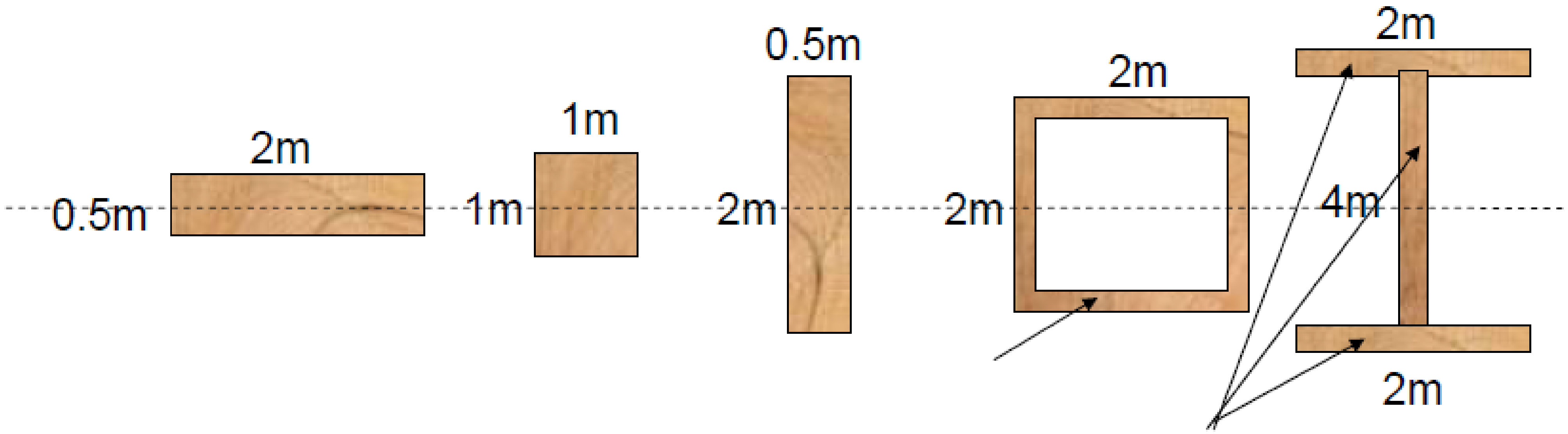
$$\int dI_y = \int_0^1 x^2 . (y . d_x)$$

$$\int dI_y = \int_0^1 x^2 . (2 - 2x^3) . d_x$$

$$\int dI_y = \int_0^1 (2x^2 - 2x^5) . d_x$$

$$I_y = \left[\frac{2x^3}{3} - \frac{2x^6}{6}\right] = \frac{1}{3}\ m^4$$

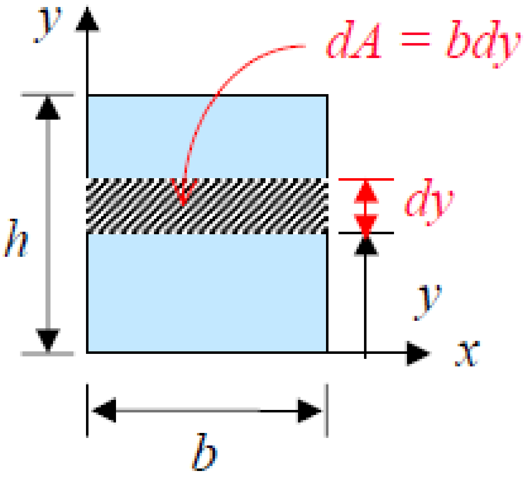
The physical meaning of moment of inertia



Area	$1m^2$	$1m^2$	$1m^2$	$1m^2$	$1m^2$
Moment of Inertia	$0.02\ m^4$	$0.08\ m^4$	$0.33m^4$	$0.55m^4$	$2.79m^4$

Moment of inertia of common area

- **Moment of Inertia of a Rectangular Area.**

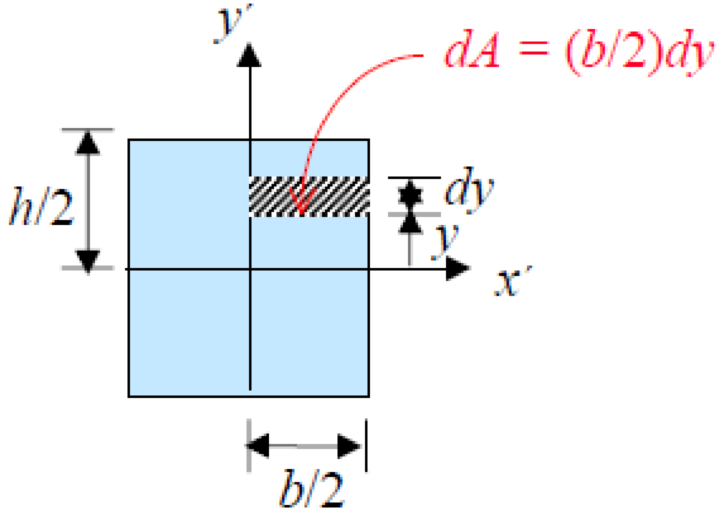


$$I_x = \int_A y^2 dA$$

$$= \int_0^h y^2 (bdy)$$

$$= \left. \frac{(by^3)}{3} \right|_0^h$$

$$= \frac{bh^3}{3}$$

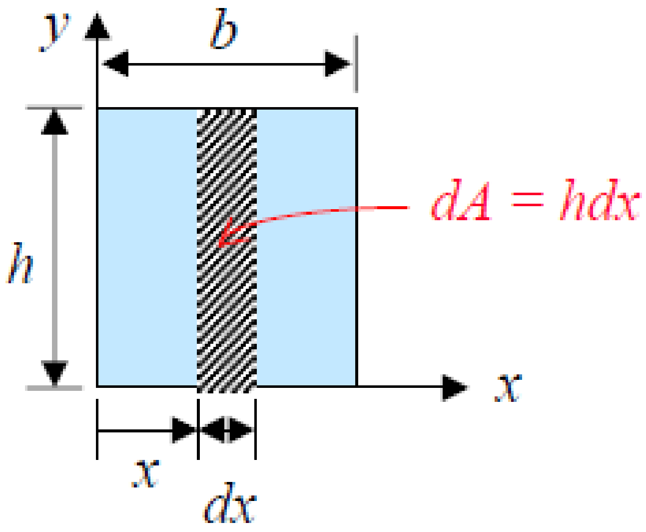


$$\bar{I}_x = I_{x'} = \int_A y^2 dA$$

$$= 4\int_0^{h/2} y^2 \left(\frac{b}{2} dy\right)$$

$$= 4\left(\frac{b}{2}\right) \left. \frac{y^3}{3} \right|_0^{h/2}$$

$$= \frac{bh^3}{12}$$

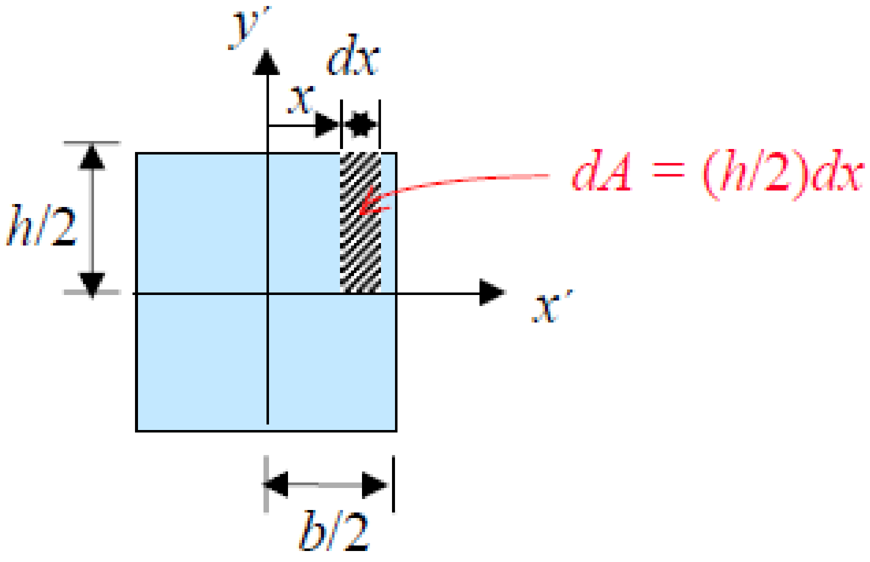


$$I_y = \int_A x^2 dA$$

$$= \int_0^b x^2 (h dx)$$

$$= \left.\frac{(hx^3)}{3}\right|_0^b$$

$$= \frac{hb^3}{3}$$



$$\bar{I}_y = I_{y'} = \int_A x^2 dA$$

$$= 4\int_0^h x^2 \left(\frac{h}{2} dx\right)$$

$$= 4\left(\frac{h}{2}\right)\left.\frac{x^3}{3}\right|_0^{b/2}$$

$$= \frac{hb^3}{12}$$

Parallel Axis Theorem:

If you know the moment of inertia about a centroid axis of a figure, you can calculate the moment of inertia about any parallel axis to the centroid axis using a simple formula;

$I_x = Ix^\circ + y^2 . A$

$I_Y = Iy^\circ + x^2 . A$

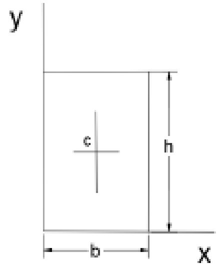
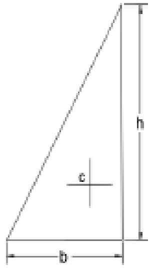
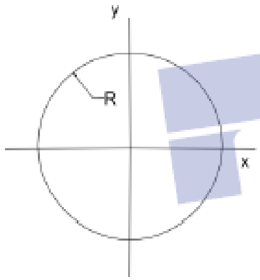
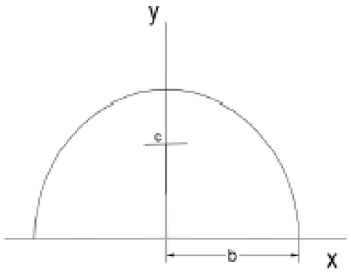
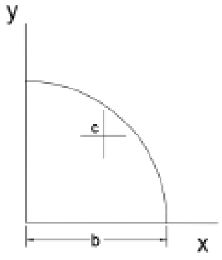
I_x : Moment of inertia about x axis.

I_Y : Moment of inertia about y axis.

Ix°: Moment of inertia about centroid axis.

Iy°: Moment of inertia about centroid axis.

y^2, x^2: distance from centroid to y,x axis's

Shape	Moment of inertia	Area and centroid
	$Ix_0 = \frac{bh^3}{12}$, $Iy_0 = \frac{hb^3}{12}$ $I_x = \frac{bh^3}{3}$, $I_y = \frac{hb^3}{3}$	$A = hb$ $x = \frac{b}{2}$, $y = \frac{h}{2}$
	$Ix_0 = \frac{bh^3}{36}$, $Iy_0 = \frac{hb^3}{36}$ $I_x = \frac{bh^3}{12}$, $I_y = \frac{hb^3}{12}$	$A = \frac{hb}{2}$ $x = \frac{b}{3}$, $y = \frac{h}{3}$
	$Ix_0 = \frac{\pi R^4}{4}$, $Iy_0 = \frac{\pi R^4}{4}$ $I_x = \frac{\pi R^4}{4}$, $I_y = \frac{\pi R^4}{4}$	$A = \pi R^2$
	$Ix_0 = Iy_0 = 0.0549R^4$ $I_x = I_y = \frac{\pi R^4}{8}$	$A = \frac{\pi R^2}{2}$ $x = \frac{4R}{3\pi}$, $y = \frac{4R}{3\pi}$
	$Ix_0 = Iy_0 = 0.1098R^4$ $I_x = I_y = \frac{\pi R^4}{16}$	$A = \frac{\pi R^2}{4}$ $x = \frac{4R}{3\pi}$, $y = \frac{4R}{3\pi}$

Example (3): Determine the moment of inertia of the shaded area in the fig. (3) With x-axis the equation of the curve is $y^2 = 4ax$ and the line $x = 9a$

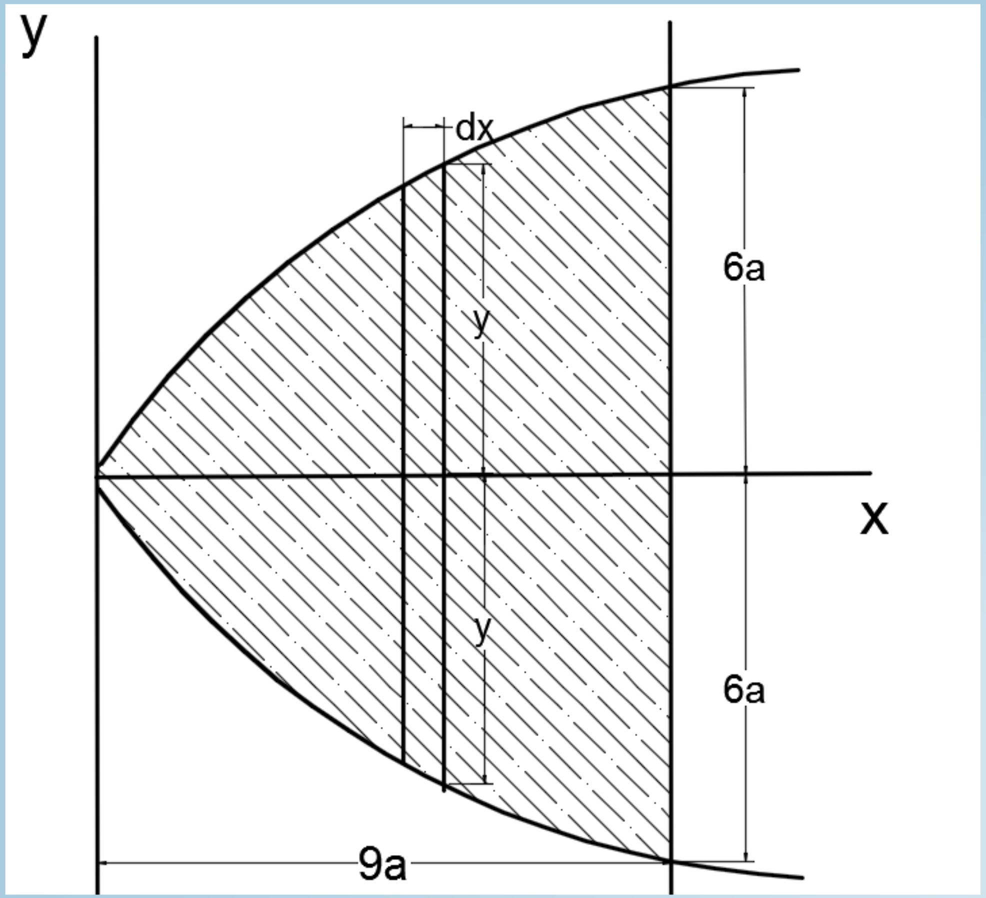


Fig. (3)

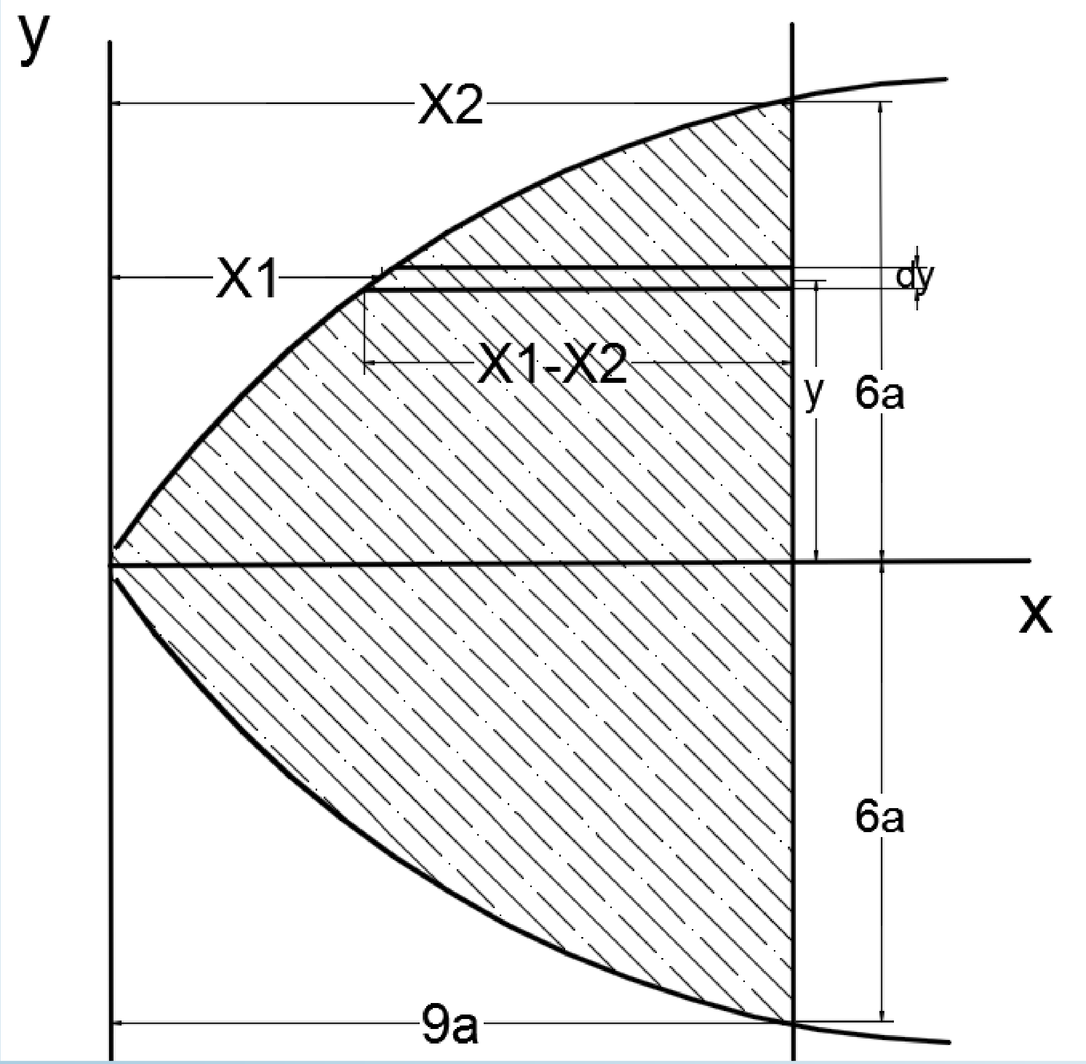
$$dI_x = \frac{hb^3}{12} = \frac{dx}{12}\,(8y^3)$$

$$y = 2a^{\frac{1}{2}}x^{\frac{1}{2}}$$

$$\int \quad dI_x = \int_0^{9a} \frac{dx}{12} 8.8a^{\frac{3}{2}}x^{\frac{3}{2}} = \int_0^{9a} \frac{16}{3} a^{\frac{3}{2}}x^{\frac{3}{2}}\,dx$$

$$I_x = \frac{16}{3}.\,a^{\frac{3}{2}}\left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right] = 518\,.4\;a^4\;m^4$$

Example (4): Determine the moment of inertia of the shaded area in the fig. (3) With x-axis the equation of the curve is $y2 = 4ax$ and the line $x = 9a$, take horizontal slice



$$dI_x = 2\int_0^{6a} y^2 . dA$$

$$dI_x = 2\int_0^{6a} y^2 . (X2 - X1). dx$$

$$dI_x = 2\int_0^{6a} y^2 . (9a - \frac{y^2}{4a}). dx$$

$$dI_x = 2\int_0^{6a} (y^2\, 9a - \frac{y^4}{4a}). dx$$

$$I_x = 2\left[\frac{9ay^3}{3} - \frac{y^5}{20a}\right] = 518.4a^4\, m^4$$

Example (5): Determine the moment of inertia of the shaded area in the fig. (4) With x-axis and y-axis.

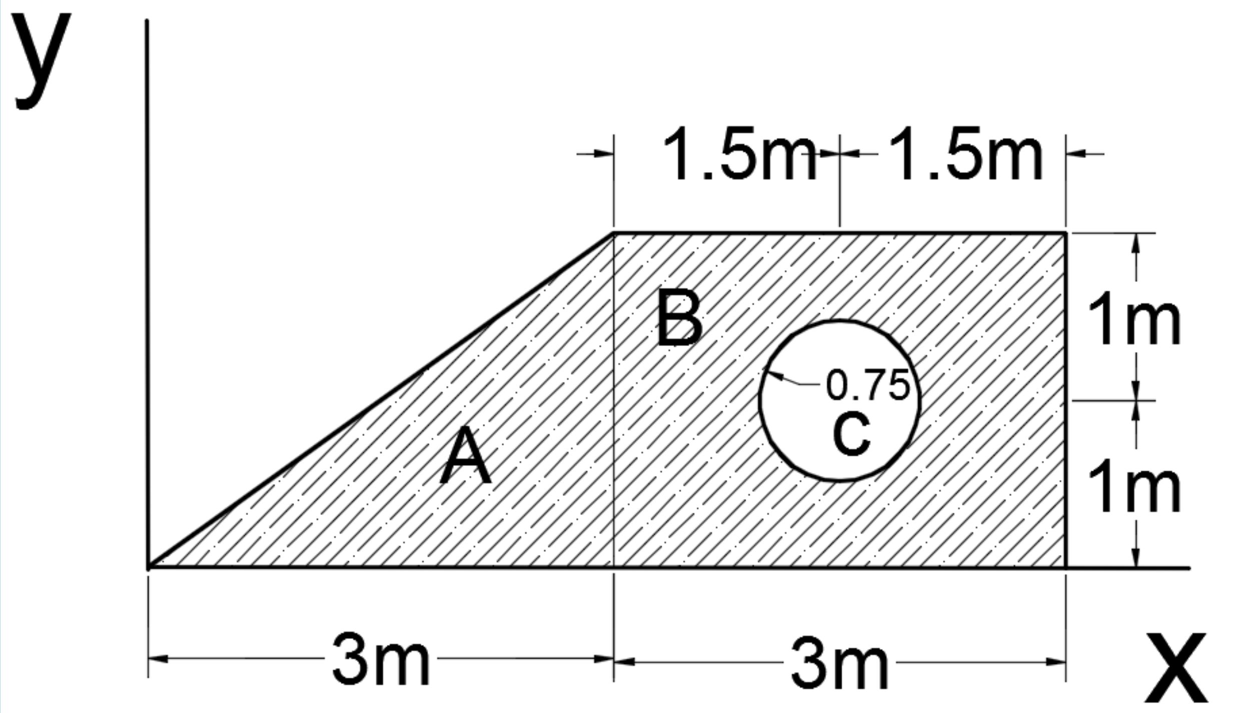


Fig. (4)

$Ix_0(m^4)$	A (m^2)	$y^2(m^2)$	A. y^2	Ix (m^4)
$A = \frac{bh^3}{36} = 0.667$	3	0.444	1.334	2
$B = \frac{bh^3}{12} = 2$	6	1	6	8
$C = \frac{\pi R^4}{4} = -0.25$	-1.76	1	-1.76	-2.01
Total				7.98

$Iy_0(m^4)$	A (m^2)	$x^2(m^2)$	A. x^2	Iy (m^4)
$A = \frac{hb^3}{36} = 1.5$	3	2^2=4	12	13.5
$B = \frac{hb^3}{12} = 4.5$	6	4.5^2=20.25	121.5	126
$C = \frac{\pi R^4}{4} = -0.25$	-1.76	4.5^2=20.25	-35.64	-35.79
Total				103.71

Example (7): Determine the moment of inertia of the shaded area in the fig. (6) With x-axis.

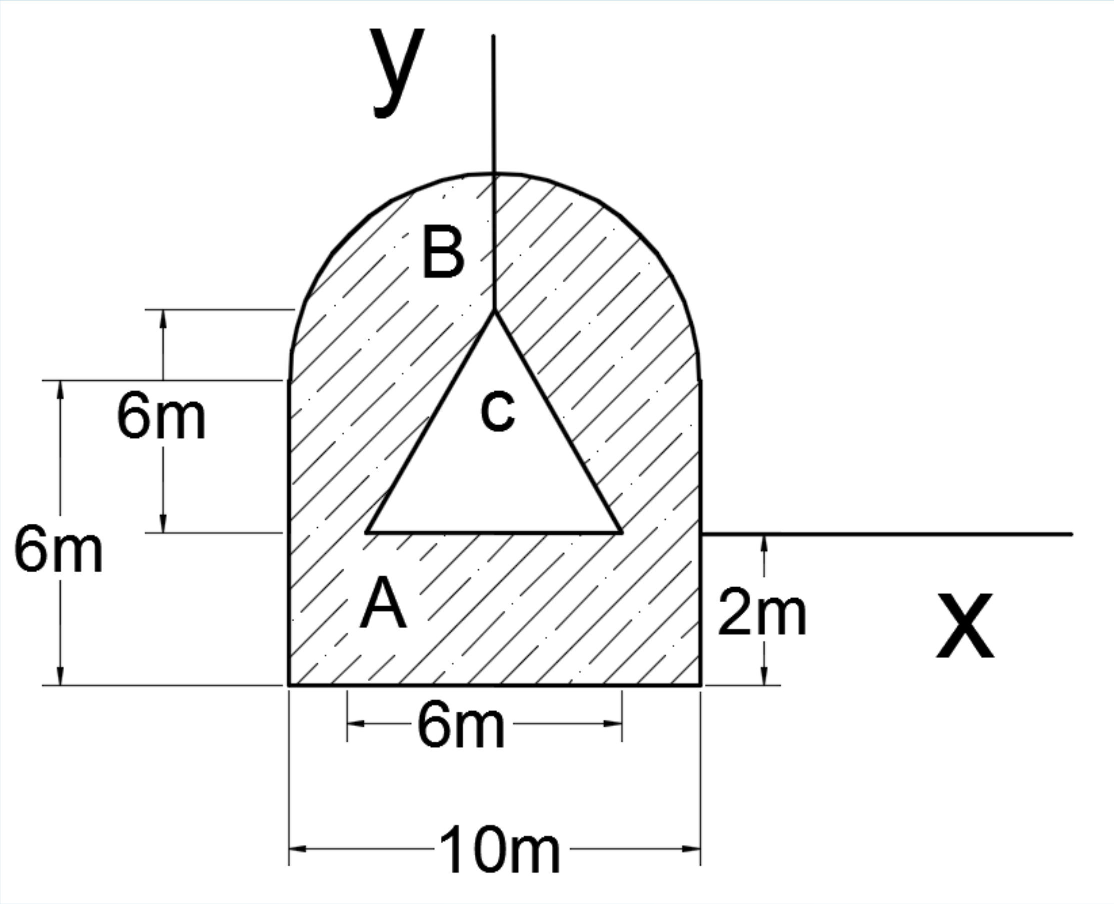


Fig. (6)

$Ix_0(m^4)$	A (m^2)	$y^2(m^2)$	A. y^2	Ix (m^4)
A $= \frac{bh^3}{12} = 180$	60	1	60	240
C = 0.1098 R^4= 68.6	39.25	37.5	37.5	1540
B $= \frac{bh^3}{36}$ = -36	-18	4	-72	-108
Total				1672

Example (8): Determine the moment of inertia of the shaded area in the fig. (7) With y-axis.

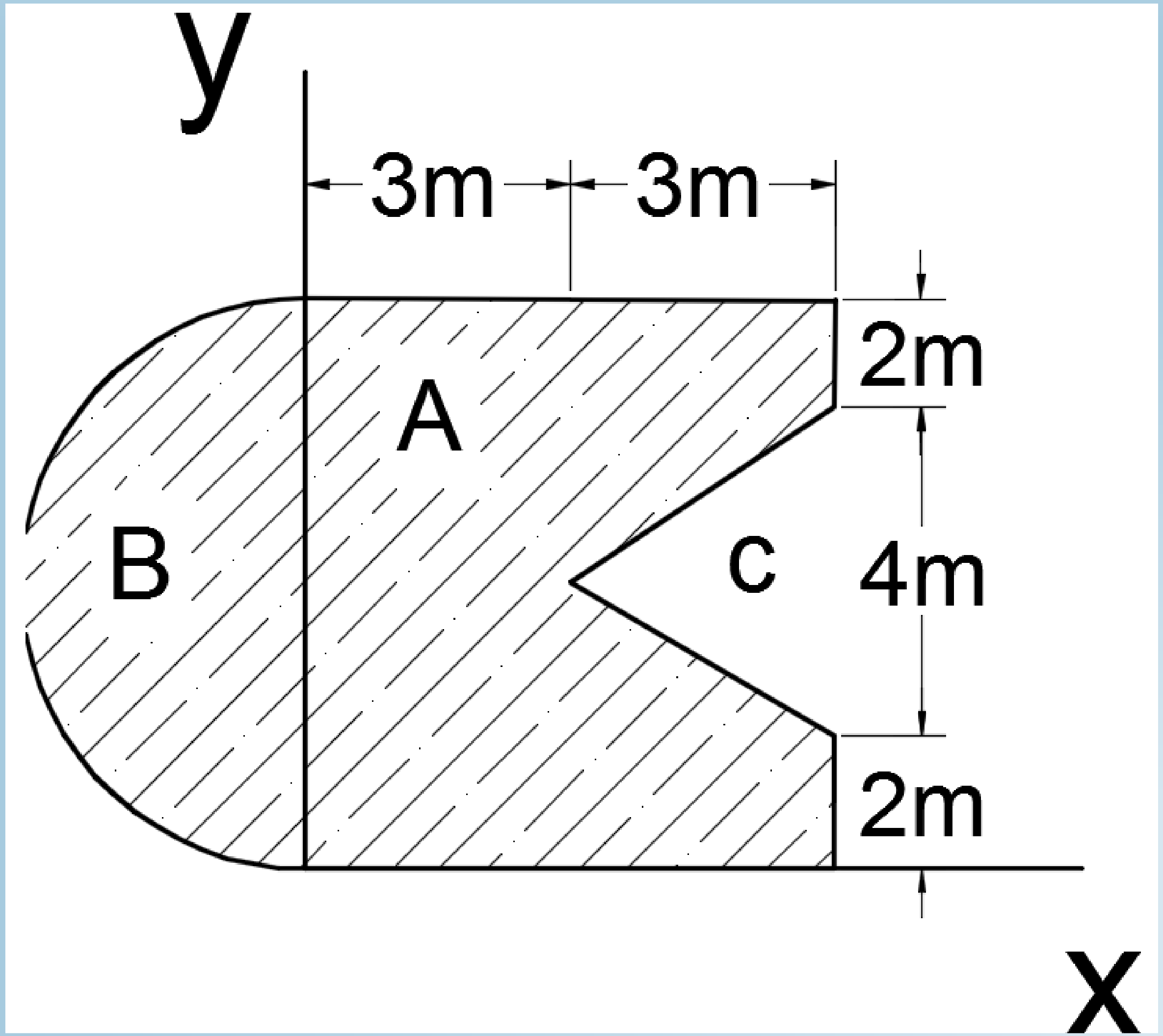


Fig. (7)

$Iy_0(m^4)$	A (m^2)	$x^2(m^2)$	A. x^2	Iy (m^4)
A $= \frac{hb^3}{12} = 144$	48	9	432	576
C $= \frac{hb^3}{36} =$ -3	-6	25	-150	-153
B $= 0.1098R^4 = 28$	25.12	2.88	72.5	100.5
Total				523.5

Thank you for listening